Published by Institute of Physics Publishing for SISSA/ISAS
Received: *January 30, 2004*
Accepted: *March 12, 2004*

Flat currents in the classical $AdS_5 \times S^5$ pure spinor superstring

Brenno Carlini Vallilo

Instituto de Física Teórica, Universidade Estadual Paulista
Rua Pamplona 145, 01405-900, São Paulo, SP, Brasil
E-mail: vallilo@ift.unesp.br

Abstract: It is proven that the classical pure spinor superstring in an $AdS_5 \times S^5$ background has a flat current depending on a continuous parameter. This generalizes the recent result of Bena, et al. for the classical Green-Schwarz superstring.

Keywords: Superstrings and Heterotic Strings, AdS-CFT and dS-CFT Correspondence.

JHEP03(2004)037

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JHEP03(2004)037

1. Introduction

One of the main difficulties of using the quantized superstring in the AdS/CFT conjecture is the sigma-model describing the dynamics of the string in AdS spaces. The RNS superstring is not apropriate to study RR backgrounds and the covariant quantization of the GS description is still an open problem. The Berkovits' pure spinor formalism overcomes these two difficulties [1]. In the pure spinor description, the $AdS_5 \times S^5$ superstring is a coset sigma model plus a WZ term, and in the large radius limit quantization can be done without kappa symmetry complications [2].

Recently, Bena et al. [3] have shown that there exist a parameter dependent flat current taking values in the PSU$(2,2|4)$ algebra in the classical GS superstring description.[1] This flat current can be used to construct an infinite number of non-local conserved charges. The existence of these charges may be a signal that the superstring sigma model might be integrable. Integrability allows the study of the sigma model outside the large radius limit. In the present work, this result will be extended to the pure spinor description of the superstring.

There are several questions to be answered before the application of the methods of integrable theories to the superstring. The first one is about the existence of these flat currents after quantization. Moreover, in general, integrable models have a mass gap and asymptotic freedom. This appears to be incompatible with the world sheet description, since the sigma model must be a conformal field theory and the spatial coordinate is compact. The first point is presently under investigation [5].

The result presented here may be easily generalized to the hybrid superstring descriptions in $AdS_2 \times S^2$ [6] and $AdS_3 \times S^3$ [7, 8]. Note that those descriptions of the superstring are also covariantly quantizable. There is also a natural extension of this work to the case

[1]See [4] for an earlier work in the bosonic case.

of pure spinor superstring in a pp-wave background [9]. Since the RR flux in that case is not an invertible matrix,[2] the world sheet fields $(d_\alpha, \widehat{d}_{\widehat{\alpha}})$ cannot be eliminated by their equations of motion and they must be included in the flat current.

This paper is organized as follows. In section 2 a short review of the pure spinor superstring is given. The construction of the flat current is done in section 3. The appendices A and B have short derivations that were omitted in the body of the paper.

2. Pure spinor superstring in an $AdS_5 \times S^5$ background

In this section I review the pure spinor formalism for the $AdS_5 \times S^5$ background as a coset supermanifold.

As was shown in [10], the $AdS_5 \times S^5$ background can be described by a coset supergroup element g taking values in PSU(2,2|4)/ SO(4,1) × SO(5) where the supervierbein and spin connections are given by

$$E^A_M dy^M = (g^{-1}dg)^A = J^A \,,$$

where $A = (\underline{a}, \alpha, \widehat{\alpha}, [\underline{ab}])$ and $\underline{a}$ signifies either a or a' and $[\underline{cd}]$ signifies either $[ab]$ or $[a'b']$, $a = 0$ to 4 and $a' = 5$ to 9. The non-vanishing structure constants f_{AB}^C of the PSU(2,2|4) algebra are

$$\begin{aligned}
f_{\alpha\beta}^{\underline{c}} &= 2\gamma_{\alpha\beta}^{\underline{c}}\,, \qquad f_{\widehat{\alpha}\widehat{\beta}}^{\underline{c}} = 2\gamma_{\alpha\beta}^{\underline{c}}\,,\\
f_{\alpha\widehat{\beta}}^{[ef]} &= f_{\widehat{\beta}\alpha}^{[ef]} = \left(\gamma^{ef}\right)_\alpha{}^\gamma \delta_{\gamma\widehat{\beta}}\,, \qquad f_{\alpha\widehat{\beta}}^{[e'f']} = f_{\widehat{\beta}\alpha}^{[e'f']} = -\left(\gamma^{e'f'}\right)_\alpha{}^\gamma \delta_{\gamma\widehat{\beta}}\,,\\
f_{\alpha\underline{c}}^{\widehat{\beta}} &= -f_{\underline{c}\alpha}^{\widehat{\beta}} = \frac{1}{2}\left(\gamma_{\underline{c}}\right)_{\alpha\beta}\delta^{\beta\widehat{\beta}}\,, \qquad f_{\widehat{\alpha}\underline{c}}^{\beta} = -f_{\underline{c}\widehat{\alpha}}^{\beta} = -\frac{1}{2}(\gamma_{\underline{c}})_{\widehat{\alpha}\widehat{\beta}}\delta^{\beta\widehat{\beta}}\,,\\
f_{cd}^{[ef]} &= \frac{1}{2}\delta_c^{[e}\delta_d^{f]}\,, \qquad f_{c'd'}^{[e'f']} = -\frac{1}{2}\delta_{c'}^{[e'}\delta_{d'}^{f']}\,,\\
f_{[\underline{cd}][\underline{ef}]}^{[\underline{gh}]} &= \frac{1}{2}\left(\eta_{\underline{ce}}\delta_{\underline{d}}^{[\underline{g}}\delta_{\underline{f}}^{\underline{h}]} - \eta_{\underline{cf}}\delta_{\underline{d}}^{[\underline{g}}\delta_{\underline{e}}^{\underline{h}]} + \eta_{\underline{df}}\delta_{\underline{c}}^{[\underline{g}}\delta_{\underline{e}}^{\underline{h}]} - \eta_{\underline{de}}\delta_{\underline{c}}^{[\underline{g}}\delta_{\underline{f}}^{\underline{h}]}\right) \qquad (2.1)\\
f_{[\underline{cd}]\underline{e}}^{\underline{f}} &= -f_{\underline{e}[\underline{cd}]}^{\underline{f}} = \eta_{\underline{e}[\underline{c}}\delta_{\underline{d}]}^{\underline{f}}\,, \quad f_{[\underline{cd}]\alpha}^{\beta} = -f_{\alpha[\underline{cd}]}^{\beta} = \frac{1}{2}(\gamma_{\underline{cd}})_\alpha{}^\beta\,, \quad f_{[\underline{cd}]\widehat{\alpha}}^{\widehat{\beta}} = -f_{\widehat{\alpha}[\underline{cd}]}^{\widehat{\beta}} = \frac{1}{2}(\gamma_{\underline{cd}})_{\widehat{\alpha}}{}^{\widehat{\beta}}\,.
\end{aligned}$$

The PSU(2,2|4) algebra $\mathcal{H}$ has a natural decomposition [6] $\mathcal{H} = \sum \mathcal{H}_i + \mathcal{H}_0$, $i = 1$ to 3

$$J_{\underline{a}} \in \mathcal{H}_2\,, \qquad J_{[\underline{ab}]} \in \mathcal{H}_0\,, \qquad J_\alpha \in \mathcal{H}_1\,, \qquad J_{\widehat{\alpha}} \in \mathcal{H}_3\,. \qquad (2.2)$$

As it can be seen from the structure constants (2.1)

$$[\mathcal{H}_0, \mathcal{H}_0] \subset \mathcal{H}_0\,, \qquad [\mathcal{H}_0, \mathcal{H}_i] \subset \mathcal{H}_i\,, \qquad [\mathcal{H}_i, \mathcal{H}_j] \subset \mathcal{H}_{i+j} \quad \mathrm{mod}\ 4\,. \qquad (2.3)$$

The bilinear form also respects the decomposition

$$\langle \mathcal{H}_0, \mathcal{H}_i \rangle = 0\,, \qquad \langle \mathcal{H}_0, \mathcal{H}_0 \rangle \neq 0\,, \qquad \langle \mathcal{H}_i, \mathcal{H}_j \rangle = 0 \qquad \text{unless} \qquad i + j = 0 \pmod 4\,. \qquad (2.4)$$

[2]The flux in the AdS background is proportional to γ^{01234}, 0, 1, 2, 3 and 4 being the AdS directions, in the pp-wave, the flux is proportional to γ^{-1234}, where $-$ is the light-cone direction.

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The action for the pure spinor superstring in this background is [1, 11, 2]

$$
\begin{aligned}
S_{\rm AdS} = & \int d^2 z \left[\langle J_2, \overline{J}_2 \rangle + \frac{3}{2} \langle J_3, \overline{J}_1 \rangle + \frac{1}{2} \langle \overline{J}_3, J_1 \rangle \right] + \\
& + \int d^2 z \left[N_{\underline{cd}} \overline{J}^{[\underline{cd}]} + \overline{N}_{\underline{cd}} J^{[\underline{cd}]} + \frac{1}{2} N_{\underline{cd}} \overline{N}^{\underline{cd}} \right] + S_\lambda + S_{\widehat{\lambda}} \,, \qquad (2.5)
\end{aligned}
$$

where $J^A = (g^{-1}\partial g)^A$ and $\overline{J}^A = (g^{-1}\overline{\partial} g)^A$ are left-invariant currents constructed from the supergroup element $g \in \mathrm{PSU}(2,2|4)$ and $J_i = J|_{\mathcal{H}_i}$. S_λ and $S_{\widehat{\lambda}}$ are the free field actions for the bosonic left and right-moving ghosts λ^α and $\widehat{\lambda}^{\widehat{\alpha}}$ satisfying the pure spinor conditions

$$
\lambda \gamma^{\underline{a}} \lambda = 0 \qquad \text{and} \qquad \widehat{\lambda} \gamma^{\underline{a}} \widehat{\lambda} = 0 \,. \qquad (2.6)
$$

Although an explicit form of S_λ and $S_{\widehat{\lambda}}$ exist, it will not be needed here. $(N^{cd}, N^{c'd'})$ and $(\overline{N}^{cd}, \overline{N}^{c'd'})$ are the $\mathrm{SO}(4,1) \times \mathrm{SO}(5)$ components of the Lorentz current for the bosonic ghosts. The OPE's of λ^α and $\widehat{\lambda}^{\widehat{\alpha}}$ with these currents are manifestly $\mathrm{SO}(4,1) \times \mathrm{SO}(5)$ covariant. Instead of having kappa symmetry, the action is BRST invariant with charges given by $Q = \oint \lambda^\alpha J_1^{\widehat{\alpha}} \delta_{\alpha\widehat{\alpha}}$ and $\overline{Q} = \oint \widehat{\lambda}^{\widehat{\alpha}} \overline{J}_3^{\alpha} \delta_{\alpha\widehat{\alpha}}$ [11]. Physical states are vertx operators in the cohomology of Q and $\overline{Q}$.

Under a local $\mathrm{SO}(4,1) \times \mathrm{SO}(5)$ transformation parametrized by $\Omega \in \mathcal{H}_0$,

$$
\begin{aligned}
\delta J_i &= [J_i, \Omega] \,, \qquad \delta \overline{J}_i = [\overline{J}_i, \Omega] \,, \qquad \delta J_0 = \partial \Omega + [J_0, \Omega] \,, \qquad \delta \overline{J}_0 = \overline{\partial} \Omega + [\overline{J}_0, \Omega] \,, \\
\delta(S_\lambda + S_{\widehat{\lambda}}) &= -\partial \Omega^{\underline{ab}} N_{\underline{ab}} - \overline{\partial} \Omega^{\underline{ab}} \overline{N}_{\underline{ab}} \,, \\
\delta N &= [N, \Omega], \quad \delta \overline{N} = [\overline{N}, \Omega] \,. \qquad (2.7)
\end{aligned}
$$

To calculate the flatness condition one must know the equations of motion that follows from (2.5). Under an arbitrary variation $\delta g = gX$, $\delta g^{-1} = -Xg^{-1}$ with $X \in \mathcal{H}_i$,

$$
\delta J = \partial X + [J, X] \,, \qquad \delta \overline{J} = \overline{\partial} X + [\overline{J}, X] \,. \qquad (2.8)
$$

Together with the Maurer-Cartan equations $\partial \overline{J} - \overline{\partial} J + [J, \overline{J}] = 0$, (2.8) implies the equations of motion

$$
\begin{aligned}
\nabla \overline{J}_2 &= -[J_3, \overline{J}_3] - \frac{1}{2}[N, \overline{J}_2] + \frac{1}{2}[J_2, \overline{N}] \,, \\
\overline{\nabla} J_2 &= [J_1, \overline{J}_1] + \frac{1}{2}[J_2, \overline{N}] - \frac{1}{2}[N, \overline{J}_2] \,, \\
\overline{\nabla} J_1 &= \frac{1}{2}[N, \overline{J}_1] - \frac{1}{2}[J_1, \overline{N}] \,, \\
\nabla \overline{J}_1 &= -[J_2, \overline{J}_3] - [J_3, \overline{J}_2] + \frac{1}{2}[N, \overline{J}_1] - \frac{1}{2}[J_1, \overline{N}] \,, \\
\nabla \overline{J}_3 &= \frac{1}{2}[N, \overline{J}_3] - \frac{1}{2}[J_3, \overline{N}] \,, \\
\overline{\nabla} J_3 &= [J_2, \overline{J}_1] + [J_1, \overline{J}_2] + \frac{1}{2}[N, \overline{J}_3] - \frac{1}{2}[J_3, \overline{N}] \,, \qquad (2.9)
\end{aligned}
$$

where $\nabla = \partial + [J_0, \;\,]$ and $\overline{\nabla} = \overline{\partial} + [\overline{J}_0, \;\,]$.

Instead of using $(\lambda, \widehat{\lambda})$ and their conjugate momenta $(\omega, \widehat{\omega})$ to derive the equation of motion for $(N, \overline{N})$, this can be done using the gauge invariance (2.7) ignoring the variation of $(J_0, \overline{J}_0)$. This is because (2.7) is the most general $\mathrm{SO}(4,1) \times \mathrm{SO}(5)$ covariant variantion of N and $\overline{N}$. The gauge symmetry is a special case. Varying the second line of (2.5) with independent Λ and $\overline{\Lambda}$ gives

$$\overline{\nabla} N = \frac{1}{2}[N, \overline{N}] \,, \qquad \nabla \overline{N} = -\frac{1}{2}[N, \overline{N}] \,. \tag{2.10}$$

The pure $AdS_5 \times S^5$ spinor superstring was first presented in the paper [1]. In [11], the massless vertex operator corresponding to supergravity fluctuations around the AdS background was studied. It was shown that the classical BRST currents $\lambda^\alpha J_1^{\widehat{\alpha}} \delta_{\alpha\widehat{\alpha}}$ and $\widehat{\lambda}^{\widehat{\alpha}} \overline{J}_3^\alpha \delta_{\alpha\widehat{\alpha}}$ are holomorphic and antiholomorphic and their corresponding BRST operators anticommute. One loop conformal invariance of (2.5) was proved in [2].

3. Flat currents in the classical pure spinor superstring

In a recent paper, Bena et al. [3] showed that the classical GS superstring action in the $AdS_5 \times S^5$ background has a one parameter dependent flat current. This flat current can be used to construct infinitely many conserved charges. The existence of these charges is an indication that the model might be integrable. Although the GS superstring is a well defined classical theory, its quantization proves to be a major problem and it is only well understood in the lighcone gauge.

In this section I will extend the results of [3] to the case of the pure spinor superstring. Under the the $\mathcal{H}_0$-gauge transformation $g \to gh$, J_0 and $\overline{J}_0$ transform as connections and J_i, N and $\overline{J}_i$, $\overline{N}$ transform in the adjoint $\mathcal{H}_0$-representation. This means that the currents gJ_ig^{-1}, gNg^{-1} and $g\overline{J}_ig^{-1}$, $g\overline{N}g^{-1}$ are invariant under the $\mathcal{H}_0$ transformation.

If it is possible to find a μ dependent $\mathcal{H}_0$ invariant current $a(\mu)$ that satisfies the flatness condition

$$da(\mu) + a(\mu) \wedge a(\mu) = 0 \,, \tag{3.1}$$

an infinit number of conserved charges can be constructed. See appendix B for a short derivation.

Since the left invariant currents J_i and $\overline{J}_i$ are easier to handle, it is useful to write (3.1) in terms of a left invariant $A(\mu) = g^{-1}a(\mu)g$. The last term in (3.1) is covariant under this transformation, but $da(\mu)$ is transformed to

$$d(g^{-1}a(\mu)g) = -g^{-1}dg \wedge (g^{-1}a(\mu)g) + g^{-1}da(\mu)g - (g^{-1}a(\mu)g) \wedge g^{-1}dg \,.$$

Writing in terms of A and $J = \sum J_i + J_0$,

$$g^{-1}da(\mu)g = dA(\mu) + J \wedge A(\mu) + A(\mu) \wedge J \,.$$

(3.1) is now writen as

$$dA(\mu) + J \wedge A(\mu) + A(\mu) \wedge J + A(\mu) \wedge A(\mu) = 0 \,. \tag{3.2}$$

In the $z, \overline{z}$ notation,

$$\partial \overline{A} - \overline{\partial} A + [A, \overline{A}] + [J, \overline{A}] + [A, \overline{J}] = 0\,. \tag{3.3}$$

One aspect that makes the pure spinor superstring quantizable is that there are equations of motion for all the J_i and $\overline{J}_i$. This enables us to construct a flat connection A in terms of all of them

$$\begin{aligned} A &= aJ_2 + bJ_1 + cJ_3\,, \\ \overline{A} &= d\overline{J}_2 + e\overline{J}_1 + f\overline{J}_3\,. \end{aligned}$$

In [3] only antisymmetric combinations of $(J_1, \overline{J}_1)$ and $(J_3, \overline{J}_3)$ could be included in $A(\mu)$. In the GS superstring kappa symmetry is an essential ingredient and restricts the form of the action. The equations of motion for $(J_1, \overline{J}_1)$ and $(J_3, \overline{J}_3)$ in the the GS model are

$$[J_2, \overline{J}_3] = -[J_3, \overline{J}_2], \quad [J_2, \overline{J}_1] = -[J_1, \overline{J}_2]\,,$$

with no terms containing covariant derivatives. Only the Maurer-Cartan identity can be used in (3.3) before fixing kappa symmetry.

The quantizable action also has the bosonic pure spinor ghosts. These ghosts have Lorentz currents N and $\overline{N}$ that interact with the sigma model, so they have to be included in A and $\overline{A}$

$$\begin{aligned} A &= aJ_2 + bJ_1 + cJ_3 + gN\,, \\ \overline{A} &= d\overline{J}_2 + e\overline{J}_1 + f\overline{J}_3 + h\overline{N}\,. \end{aligned} \tag{3.4}$$

Now (3.3) is expanded using A and $\overline{A}$ given above.

$$\begin{aligned} &d\nabla\overline{J}_2 + e\nabla\overline{J}_1 + f\nabla\overline{J}_3 + h\nabla\overline{N} - a\overline{\nabla}J_2 - b\overline{\nabla}J_1 - c\overline{\nabla}J_3 - g\overline{\nabla}N + \\ &+(ad+d+a)[J_2,\overline{J}_2] + (ae+e+a)[J_2,\overline{J}_1] + (af+f+a)[J_2,\overline{J}_3] + \\ &+(bd+d+b)[J_1,\overline{J}_2] + \\ &+(be+b+e)[J_1,\overline{J}_1] + (bf+b+f)[J_1,\overline{J}_3] + \\ &+(cd+c+d)[J_3,\overline{J}_2] + (ce+c+e)[J_3,\overline{J}_1] + \\ &+(cf+c+f)[J_3,\overline{J}_3] + (ha+h)[J_2,\overline{N}] + (hb+h)[J_1,\overline{N}] + (hc+h)[J_3,\overline{N}] + \\ &+(gd+g)[N,\overline{J}_2] + (ge+g)[N,\overline{J}_1] + (gf+g)[N,\overline{J}_3] + gh[N,\overline{N}] = 0\,, \end{aligned} \tag{3.5}$$

Inserting the equations of motion (2.9) in (3.5), the following system of equations involving all coefficients (a, b, c, d, e, f, g, h) has to be solved

$$\begin{aligned} &ad+d+a = 0\,, && ea+e+a-c = 0\,, && af+f+a-e = 0\,, \\ &bd+d+b-c = 0\,, && be+b+e-a = 0\,, && bf+b+f = 0\,, \\ &cd+d+c-e = 0\,, && ce+c+e = 0\,, && cf+c+f-d = 0\,, \\ &gh - \frac{1}{2}(g+h) = 0\,, && ha+h-\frac{1}{2}(a-d) = 0\,, && hb+h-\frac{1}{2}(b-e)\,, \\ &hc+h-\frac{1}{2}(c-f) = 0\,, && gd+g-\frac{1}{2}(d-a) = 0\,, && ge+g-\frac{1}{2}(e-b) = 0\,, \\ & && gf+g-\frac{1}{2}(f-c) = 0\,. && \end{aligned} \tag{3.6}$$

The solution is given by

$$\begin{aligned} a &= \mu - 1\,, \qquad b = \pm(\mu)^{3/2} - 1\,, \qquad c = \pm(\mu)^{1/2} - 1\,, \\ d &= \mu^{-1} - 1\,, \qquad e = \pm(\mu)^{-1/2} - 1\,, \qquad f = \pm(\mu)^{-3/2} - 1\,, \\ g &= \frac{1}{2}(1-\mu^2)\,, \qquad h = \frac{1}{2}(\mu^{-2} - 1)\,, \end{aligned} \tag{3.7}$$

which is analytic for $\mu > 0$. It is interesting to note that the system has the same solution if we exclude the ghost currents. This is not surprising, since the ghosts are necessary to quantization and the calculation here is classical. Note that this independence means that exactly the same result holds for the hybrid superstring description in $AdS_2 \times S^2$ and $AdS_3 \times S^3$. When quantum corrections are taken into account, the ghost part has an essential role [5].

To obtain the result of Bena et al. it must be remembered that the pure spinor superstring action (2.5) can be writen in the form

$$S_{\rm PS} = S_{\rm GS} + \int d^2z(d_1\overline{J}_3 + \overline{d}_3 J_1 + d_1\overline{d}_3) + S_{\rm ghosts}$$

with the indexes contracted in a PSU$(2,2|4)$ covariant way. The GS superstring is recovered with $d_1 = 0$ and $\overline{d}_3 = 0$ and $S_{ghosts} = 0$. Calculating the equations of motion, this is equivalent to $\overline{J}_3 = 0$ and $J_1 = 0$. Doing that, we end up with the equations

$$ad + d + a = 0\,, \qquad ae + a + e - c = 0\,, \qquad cd + c + d - e = 0\,, \qquad ce + c + e = 0\,. \tag{3.8}$$

Writing (a, c, d, e) in terms of $(\alpha, \beta, \gamma, \delta)$,

$$a = \beta - \alpha, \quad c = \delta - \gamma\,, \qquad d = -\beta - \alpha, \quad e = -\gamma - \delta\,,$$

(3.8) is

$$\begin{aligned} \alpha^2 - \beta^2 - 2\alpha &= 0\,, \qquad -\beta\gamma + \alpha\gamma - \beta\delta + \alpha\gamma + \beta - \alpha - 2\delta = 0 \\ \gamma^2 - \delta^2 - 2\gamma &= 0\,, \qquad -\beta\delta + \beta\gamma + \gamma\alpha - \delta\alpha - \alpha - \beta + 2\delta = 0\,, \end{aligned} \tag{3.9}$$

which are linear combinations of their main equations.

Acknowledgments

I would like to thank L.A. Ferreira for useful discussions and especially Nathan Berkovits for useful discussions and suggestions. I also thank Dáfni Marchioro for a careful reading of the manuscript. This work was supported by FAPESP grant 00/02230-3.

A. $AdS_5 \times S^5$ string action

The action (2.5) is derived from a general curved background

$$\begin{aligned} S_{\rm curv} = \int d^2z \Big[&\frac{1}{2}(G_{MN} + B_{MN})\partial y^M \overline{\partial} y^N + \\ &+ d_\alpha E^\alpha_M \overline{\partial} y^M + \frac{1}{2} N_{mn} \overline{\partial} y^M \Omega^{mn}_M + \\ &+ \widehat{d}_{\widehat{\alpha}} \widehat{E}^{\widehat{\alpha}}_M \partial y^M + \frac{1}{2}\overline{N}_{mn}\partial y^M \widehat{\Omega}^{mn}_M + d_\alpha \widehat{d}_{\widehat{\gamma}} P^{\alpha\widehat{\gamma}} + \overline{N}_{mn} d_\alpha C^{\alpha mn} + \\ &+ N_{mn}\widehat{d}_{\widehat{\alpha}}\widehat{C}^{\widehat{\alpha} mn} + \frac{1}{2} N_{mn}\overline{N}_{op} R^{mnop} + \Phi(x, \theta, \widehat{\theta})R \Big] + S_\lambda + S_{\widehat{\lambda}}\,, \end{aligned} \tag{A.1}$$

where the superfields $(G_{MN}, B_{MN}, E_M^\alpha, \widehat{E}_M^{\widehat{\alpha}}, \Omega_M^{mn}, \widehat{\Omega}_M^{mn}, P^{\alpha\widehat{\gamma}}, C^{\alpha mn}, \widehat{C}^{\widehat{\alpha}mn}, R^{mnop}, \Phi)$ are related to the supergravity multiplet [12]. The first line in (A.1) is just the GS action in a curved background, the second and third lines are necessary to covariantly quantize the superstring, i.e., they provide invertible propagators (containing no operators that might have zero modes) for the fermions. In flat space, the world sheet fields $(d_\alpha, \widehat{d}_{\widehat{\alpha}})$ are expressed in terms of $(x, \theta, \widehat{\theta})$, but in a general background they are interpreted as independent fields. $\Phi(x, \theta, \widehat{\theta})R$ is the Fradkin-Tseytlin term, where Φ is the compensator scalar superfield whose lowest component is the dilaton and R is the worldsheet curvature. Since the dilaton is constant in the $AdS_5 \times S^5$ background, the Fradkin-Tseytlin term is integrated to give the usual genus counting coupling constant.

To get (2.5), one must know the value of the background superfields B_{AB} and $P^{\alpha\widehat{\beta}}$

$$B_{\alpha\widehat{\beta}} = B_{\widehat{\beta}\alpha} = -\frac{1}{2}(Ng_s)^{1/4}\delta_{\alpha\widehat{\beta}}, \qquad P^{\alpha\widehat{\beta}} = \frac{1}{(Ng_s)^{1/4}}\delta^{\alpha\widehat{\beta}}\,, \tag{A.2}$$

where N is the value of the Ramond-Ramond flux, g_s is the string coupling constant and $\delta_{\alpha\widehat{\beta}} = (\gamma^{01234})_{\alpha\widehat{\beta}}$ with 01234 being the directions of AdS_5. The background field $P^{\alpha\widehat{\beta}}$ makes $(d_\alpha, \widehat{d}_{\widehat{\alpha}})$ auxiliary fields which can be eliminated by their equations of motion.

B. Conserved charges from flat currents

In this appendix, a short derivation of the conserved charge implied by the flatness condition is given.[3]

It folows by the non-abelian Stokes theorem that the path ordered integral is

$$U_\omega[a] = \mathcal{P}e^{\oint_\omega a(\mu)} = 1\,, \tag{B.1}$$

for any contractible loop ω. In radial quantization, time evolution is represented by the radial coordinate. The origin is the infinite past, where usually there is some vertex operator inserted. Because of this operator insertion, loops around the origin are not contractible. The path ordered integral $\mathcal{P}e^{\oint_\rho a(\mu)}$ satisfies the group product rule,

$$U_\rho[a]U_\sigma[a] = U_{\sigma\circ\rho}[a]\,,$$

if the end of σ is the begginning of ρ.

Choosing the path in figure 1,

$$U_\alpha[a]U_\delta[a]U_\beta[a]U_\gamma[a] = U_{\gamma\circ\beta\circ\delta\circ\alpha}[a] = 1\,.$$

From this equation follows that

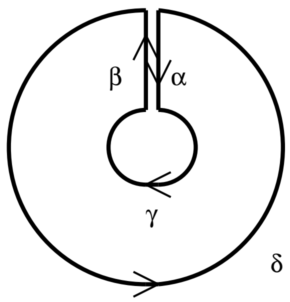

Figure 1: Path chosed to prove conservation of $\mathrm{Tr}(U_\gamma[a])$ for non-contractible loops.

$$\mathrm{Tr}(U_\gamma[a]) = \mathrm{Tr}(U_{\delta^{-1}}[a])\,, \tag{B.2}$$

which means that $\mathrm{Tr}(U_\gamma[a])$ is a conserved charge. Since the flat current a depends on a continuous parameter, there are an infinite number of conserved charges.

[3] See, for example, [13] for a review.

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